

# New results on the limit for the width of the exotic $\Theta^+$ resonance

A. Sibirtsev

*Institut für Kernphysik, Forschungszentrum Jülich, D-52425 Jülich, Germany  
Special Research Centre for the Subatomic Structure of Matter (CSSM) and Department  
of Physics and Mathematical Physics, University of Adelaide, SA 5005, Australia*

J. Haidenbauer, S. Krewald

*Institut für Kernphysik, Forschungszentrum Jülich, D-52425 Jülich, Germany*

Ulf-G. Meißner

*Universität Bonn, Helmholtz-Institut für Strahlen- und Kernphysik, Nußallee 14-16, D-53115 Bonn, Germany  
Institut für Kernphysik, Forschungszentrum Jülich, D-52425 Jülich, Germany*

(Dated: February 1, 2008)

We investigate the impact of the  $\Theta^+(1540)$  resonance on differential and integrated cross sections for the reaction  $K^+d \rightarrow K^0pp$ , where experimental information is available at kaon momenta below 640 MeV/c. The calculation utilizes the Jülich  $KN$  model and extensions of it that include contributions from a  $\Theta^+(1540)$  state with different widths. The evaluation of the reaction  $K^+d \rightarrow K^0pp$  takes into account effects due to the Fermi motion of the nucleons within the deuteron and the final three-body kinematics. We conclude that the available data constrain the width of the  $\Theta^+(1540)$  to be less than 1 MeV.

PACS numbers:

Evidence for a narrow baryon resonance with positive strangeness, the  $\Theta^+$ , has been found by more than ten experimental collaborations with masses ranging from 1521 to 1555 MeV and widths from 9 and 24 MeV (these widths are usually upper limits given by the respective detector resolution) [1]. The width of the  $\Theta^+$  is of particular importance to understand the nature of this state, see e.g. [2, 3]. Constraints for the width of the  $\Theta^+$  resonance can be deduced from  $K^+N$  and  $K^+d$  data which are available in the relevant momentum range,  $417 < k_0 < 476$  MeV/c, of the incident kaons [4, 5, 6, 7, 8, 9].

A first, rough estimate for the width of the  $\Theta^+$  based on  $K^+d$  data was given by Nussinov [10]. Assuming that there are fluctuations of about 2-4 mb in the experimental total  $K^+d$  cross section at momenta  $500 \leq k_0 \leq 700$  MeV that could be due to the  $\Theta^+$ , he deduced an upper limit of  $\Gamma_{\Theta} < 6$  MeV.

A more refined estimate based on the  $K^+d$  reaction was presented in Ref. [11]. Using data on the  $K^+d \rightarrow K^0pp$  total cross section for kaon momenta in the vicinity of the expected  $\Theta^+$  a conservative limit of 1 mb for the resonance cross section was deduced and that led them to an upper limit for the  $\Theta^+$  width of around 1.1 MeV. A similar consideration for the total  $K^+d$  cross section resulted in a limit of  $\Gamma_{\Theta} = 0.8$  MeV. Indeed, even for the certainly unrealistic assumption that the entire  $I=0$  total cross section at  $k_0 = 440$  MeV/c is due to the  $\Theta^+$  resonance, the width deduced by these authors for the  $\Theta^+$  did not exceed  $\Gamma_{\Theta} = 3.6$  MeV [11]. A similar limit ( $\Gamma_{\Theta} = 0.9$  MeV) based on the total  $K^+d$  cross section was also derived in Ref. [12] using, however, only a selected set of data.

The reexamination of the existing  $KN$  data base in

terms of a partial wave analysis, performed by Arndt, Strakovsky and Workman, led to the claim that widths of the  $\Theta^+$  larger than a few MeV are excluded [13]. Specifically, it was found that the inclusion of a  $\Theta^+$  resonance state with a width of 5 MeV in the  $P_{01}$ ,  $S_{01}$  or  $P_{03}$  partial waves resulted in an increase of the total  $\chi^2$  by 30% or more.

Similar conclusions were drawn from a direct comparison [14] between the available data on total  $KN$  cross sections in the  $I=0$  and  $I=1$  isospin channels and a  $KN$  model calculation based on the meson-exchange model of the Jülich group [17, 18]. It was argued that the rather strong enhancement of the cross section caused by the presence of a  $\Theta^+$  with a width of 20 MeV is not compatible with the existing information on  $KN$  scattering. Only a much narrower  $\Theta^+$  state, with a width in the order of 5 MeV or less, could be reconciled with the existing data base, cf. Ref. [14] – or, alternatively, the  $\Theta^+$  state must lie at an energy much closer to the  $KN$  threshold.

In this paper we want to extend the work of Ref. [14]. We perform direct comparison of a calculation of the reaction  $K^+d \rightarrow K^0pp$  with the corresponding experimental information. One has to keep in mind that the information on the  $K^+N$  interaction in the isospin  $I = 0$  channel has been inferred from data on the  $K^+d$  reaction. In the extraction procedure it is implicitly assumed that the  $K^+N$  amplitude shows no sharp structure. Therefore, it is more conclusive to calculate explicitly observables for the reactions  $K^+d \rightarrow K^+np$  and  $K^+d \rightarrow K^0pp$  based on  $K^+N$  interaction models that include a  $\Theta^+(1540)$  resonance so that a direct comparison with experimental data is possible. Then “medium” effects such as the broadening of the resonance by the Fermi motion of the nucleons in the deuteron and the interaction of the nucleons in the

final state can be dealt with rigorously.

Starting point of the present investigation is again the Jülich meson-exchange model for the  $KN$  interaction. An extensive description of this model is given in Refs. [17, 18] where one can also find its results for  $KN$  phase shifts and for cross sections and polarizations. Evidently this model yields a good overall reproduction of all presently available empirical information on  $KN$  scattering. Specifically, it describes the data up to beam momenta of  $k_0 \approx 1$  GeV/c, i.e. well beyond the region of the observed  $\Theta^+$  resonance structure [1]. Note that the parameters of the model are fixed by a simultaneous fit to all  $KN$  partial waves and therefore the contributions to the  $P_{01}$  channel, where the  $\Theta^+$  pentaquark state is supposed to occur, are constrained by the empirical information in the other partial waves. We also utilize here the variants that were presented in Ref. [14], where a  $\Theta^+$  resonance was added to the Jülich  $KN$  model with a resonance position at 1540 MeV and dynamically generated widths of 5 and 20 MeV, respectively, and we consider two more variants with widths of 1 and 10 MeV, constructed in the same way as described in Ref. [14].

When adding a new ingredient, in the form of the  $\Theta^+$  resonance, to the  $KN$  model of the Jülich group one should, in principle, refit all the free parameters of this model. However, in practice it turned out that the available experimental information (i) on those  $KN$  partial-wave amplitudes where the  $\Theta^+$  does not contribute (i.e. all except the  $P_{01}$ ) and (ii) on the behaviour of the  $P_{01}$  amplitude at higher energies, i.e. away from the  $\Theta^+$  resonance region, provides rather strong constraints on the model parameters and therefore even a very moderate change in those parameters would already lead to a deterioration of the overall description of the  $KN$  data. Moreover, one has to keep in mind that the magnitude of the  $KN$  cross section generated by a resonance is determined by unitarity constraints only, for energies below the inelastic threshold [10]. It cannot be changed by varying the parameters of the model, anyway. Let us also mention that the  $\Theta^+$  resonance is added to the  $KN$  model on the potential level, cf. Refs. [14, 15] for details. In this way ambiguities with regard to the relative phase are avoided and the interference pattern follows directly from the underlying dynamics as discussed in Ref. [15].

For the calculation of the reaction  $K^+d \rightarrow K^0pp$  we follow, in general, the theoretical procedure which was originally developed by Stenger et al. [5]. A detailed description of the formalism can be found in Ref. [16].

The amplitude  $T_d$  for the deuteron breakup reaction is

$$T_d = \sqrt{16\pi^3 m_d} [T_N(q)u(p) + T_N(p)u(q)], \quad (1)$$

where  $m_d$  is the deuteron mass,  $p$  and  $q$  are the momenta of the two final nucleons,  $u$  is the ( $S$ -wave) deuteron wave function and  $T_N$  is elementary  $KN$  amplitude. In the present calculation we used the deuteron wave function of the CD-Bonn potential. Exploratory calculations based on other wave functions (Paris, Hulthen) indicated, however, that the results are rather insensitive to the specific

choice. Note that throughout we neglect the deuteron D-state. The differential cross section for the  $K^+d \rightarrow K^0pp$  reaction is then given as [5, 16]

$$\frac{d\sigma}{d\Omega} = (|f_x|^2 + |g_x|^2)[I(\theta) - J(\theta)] + \frac{2}{3}|g_x|^2 J(\theta), \quad (2)$$

where  $f_x$  and  $g_x$  are the elementary spin-non-flip and spin-flip  $K^+n \rightarrow K^0p$  amplitudes and  $I$  and  $J$  are the deuteron inelastic form factors, respectively. They are explicitly given by [5, 16]

$$I = F \int \frac{k^2 dk}{E_K} \frac{d^3 p}{E_p} \frac{d^3 q}{E_q} \delta^4(k_0 + P - k - p - q) \frac{u^2(p) + u^2(q)}{2},$$

$$J = F \int \frac{k^2 dk}{E_K} \frac{d^3 p}{E_p} \frac{d^3 q}{E_q} \delta^4(k_0 + P - k - p - q) u(p) u(q), \quad (3)$$

where  $k_0$ ,  $k$  and  $P$  are the momenta of the initial and final kaon and of the deuteron, respectively.  $E_k$ ,  $E_p$  and  $E_q$  are the total energies of particles in the final state. The factor  $F$  accounts for the transformation of the kaon scattering angle  $\theta$  from the laboratory deuteron rest frame to the center-of-mass frame of the  $KN$  two-body system. It is usually evaluated in the stationary spectator configuration, i.e. by assuming that the reaction takes place on the neutron at rest.

In the derivation of the expression for the  $K^+d \rightarrow K^0pp$  differential cross section one encounters three-body phase space integrals of the  $KN$  amplitudes over the momentum distribution of the nucleons within the deuteron. The form factor approximation rests on the assumption that the elementary amplitudes  $f_x$  and  $g_x$  and the kinematic factors vary only slightly over the integration range and therefore can be taken out of the integrands and evaluated for a fixed typical nucleon momentum. The remaining integrals are then the form factors  $I$  and  $J$ . However, in the presence of the  $\Theta^+$  resonance the amplitudes  $f_x$  and  $g_x$  depend strongly on the kaon energy and can not be removed from the  $I$  and  $J$  integrands. Therefore, in our analysis we integrate the  $KN$  amplitude over the final three-body phase space. Furthermore we do not use the stationary spectator approximation but compare our calculations directly to the differential cross sections measured in the deuteron rest frame.

It is worthwhile to mention that for the  $K^+d \rightarrow K^0pp$  three-body final-state the invariant mass  $m_{Kp}$  of the  $K^0p$  system is integrated over the range from  $m_K + m_p$  to  $\sqrt{s} - m_p$ , where  $m_K$  and  $m_p$  stand for the masses of the  $K^0$ -meson and the proton, respectively, and

$$s = m_K^2 + m_d^2 + 2m_d \sqrt{m_k^2 + k_0^2}. \quad (4)$$

Therefore, for a fixed initial kaon momentum,  $k_0$ , the deuteron experiment samples the elementary  $K^+n \rightarrow K^0p$  amplitude over the  $m_{Kp}$  range given above. This situation substantially differs from the “free neutron target” approximation, where the invariant mass  $m_{Kp}$  is fixed by  $k_0$  through Eq. (4). For the “free target” measurements the  $\Theta^+$  mass of 1530 MeV corresponds to incident kaon

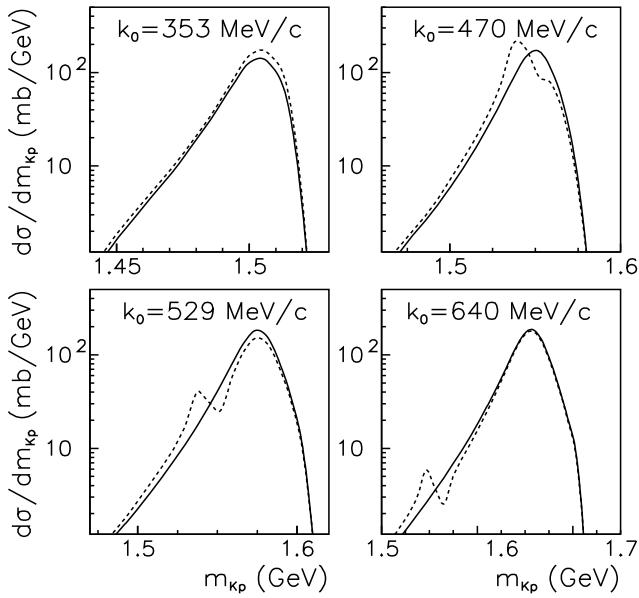


FIG. 1: The  $K^0p$  invariant mass spectra from  $K^+d \rightarrow K^0pp$  reaction at different  $K^+$ -meson momenta. Solid lines show our calculations without a  $\Theta^+(1540)$  resonance, while the dashed lines indicate the results with  $\Gamma_\Theta=5$  MeV.

momentum of  $k_0=417$  MeV/c and only the data around that momentum can be sensitive to the  $\Theta^+$  resonance. If one does not make the assumption of a free target, all  $K^+d \rightarrow K^0pp$  observables above  $k_0=417$  MeV/c will be influenced by the presence of the  $\Theta^+$  resonance, which will show up in the  $K^0p$  mass distribution. Note that the  $m_{Kp}$  spectrum is affected by the deuteron wave function, since the maximal  $K^0p$  mass corresponds to the minimal spectator momentum, while the minimal  $m_{Kp}$  probes high spectator momenta. In addition the  $\Theta^+$  resonance occupies only a small fraction of the  $K^0p$  mass distribution, while the overall  $m_{Kp}$  integration includes large part of the “non-resonant background”. Therefore it might be that the  $\Theta^+$  signal in the invariant  $K^0p$  mass spectrum becomes invisible after  $m_{Kp}$  integration. The arguments given above are confirmed by our explicit calculations of the  $K^0p$  mass spectra for different kaon momenta, which are shown in Fig. 1. Here the solid lines show the calculations without  $\Theta^+$  resonance, while the dashed lines are our results with a  $\Theta^+(1540)$  with a width  $\Gamma_\Theta=5$  MeV.

The  $K^0$ -meson angular spectra for the reaction  $K^+d \rightarrow K^0pp$  at different  $K^+$ -meson momenta are shown in Figs. 2, 3, and 4. The lines correspond to calculations with the original Jülich  $KN$  model (i.e. without a  $\Theta^+$ ) and with the variants with different  $\Theta^+$  widths. Note that the resonance energy is always the same and was assumed to be 1540 MeV. The spin and parity is assigned as  $1/2^+$ . Our results are compared with the available experimental spectra [4, 6, 7, 8] that have kaon momenta below 640 MeV/c. It is interesting to see that the calculations with a  $\Theta^+$  increase the differential  $K^+d \rightarrow K^0pp$

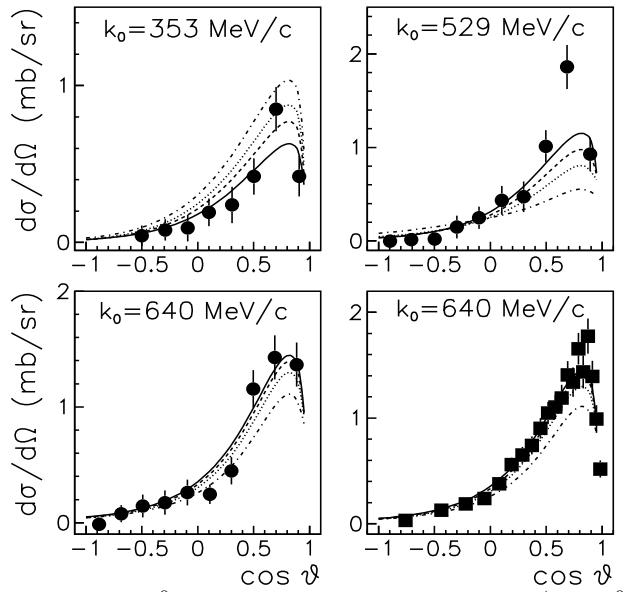


FIG. 2: The  $K^0$ -meson angular spectra from the  $K^+d \rightarrow K^0pp$  reaction for different  $K^+$  momenta. The curves show our results for the original Jülich  $KN$  model without the  $\Theta^+$  resonance (solid line) and the variants with a  $\Theta^+(1540)$  and with different widths ( $\Gamma_\Theta=5$  MeV - dashed; 10 MeV - dotted; and 20 MeV - dash-dotted). The data are from Slater et al. [6] (circles) and Giacomelli et al. [7] (squares).

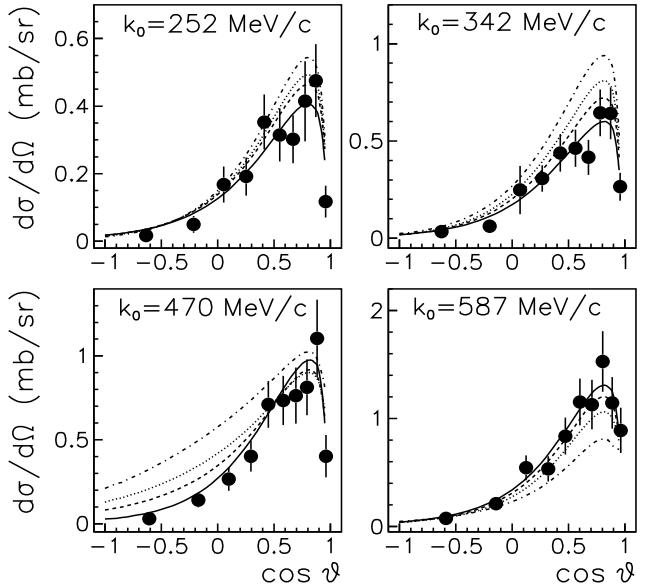


FIG. 3: The  $K^0$ -meson angular spectra from  $K^+d \rightarrow K^0pp$  reaction at different  $K^+$  momenta. For notations, see Fig. 2. The data are from Glasser et al. [8].

cross sections at momenta  $k_0 \leq 470$  MeV/c and decrease them at higher momenta as compared to the those obtained without a  $\Theta^+$  resonance. This effect is caused by the interference of the  $\Theta^+$  and the non-resonant  $P_{01}$  contribution and is clearly illustrated by the  $K^0p$  invariant mass distribution shown in Fig. 1. Note also that

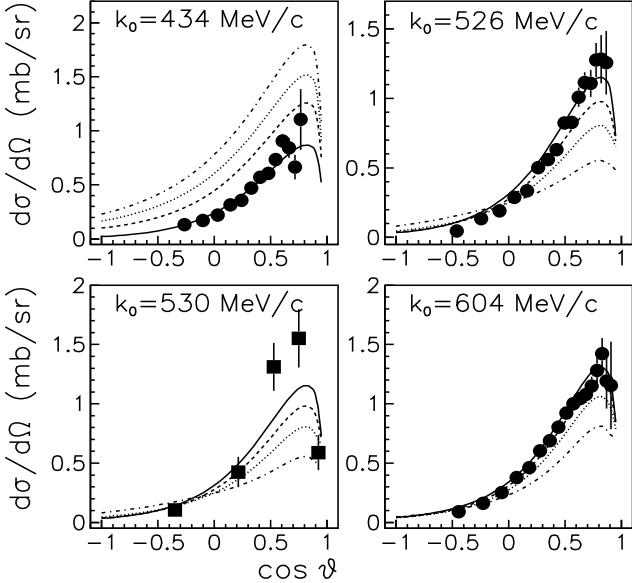


FIG. 4: The  $K^0$ -meson angular spectra from  $K^+d \rightarrow K^0pp$  reaction at different  $K^+$  momenta. For notations, see Fig. 2. The data from Damerell et al. [4] (circles) and Stenger et al. [5] (squares).

at large kaon momenta the integration over the  $K^0p$  invariant mass does not allow to distinguish between the situation with  $\Gamma_\Theta \leq 10$  MeV and that without a  $\Theta^+$  resonance.

A detailed inspection of the available differential  $K^+d \rightarrow K^0pp$  cross sections clearly indicates that the measurement by Glasser et al. [8] at the  $K^+$ -meson momentum  $k_0 = 470$  MeV/c and of Damerell et al. [4] at  $k_0 = 434$  MeV/c are the most crucial ones for the determination of the  $\Theta^+$  width. By comparing our results with the 138 experimental points on differential  $K^+d \rightarrow K^0pp$  cross sections shown in Figs. 2, 3 and 4 we can deduce a  $\chi^2/\text{dof}$ . Table 1 lists the  $\chi^2/\text{dof}$  evaluated for different  $\Theta^+$  widths. To emphasize the impact of the spectra measured at  $k_0 = 434$  MeV/c [4] and  $k_0 = 470$  MeV/c [8] we also present the  $\chi^2$  obtained by excluding them from the analysis, which is indicated as solution *B* in Table 1.

TABLE I:  $\chi^2/\text{dof}$  evaluated by comparing our calculations for different  $\Theta^+$  widths,  $\Gamma_\Theta$ , with the experimental information on  $K^0$  angular spectra measured in the reaction  $K^+d \rightarrow K^0pp$  at kaon momenta from 252 to 640 MeV/c (138 data points). Result *A* was obtained by analyzing the data shown in Figs. 2, 3 and 4. Result *B* was obtained by excluding the  $K^0$  spectra at  $k_0 = 434$  MeV/c [4] and  $k_0 = 470$  MeV/c [8].

$\Gamma_\Theta$ (MeV)	0	5	10	20
<i>A</i>	1.8	7.4	27.7	42.2
<i>B</i>	1.4	1.7	2.4	4.8

In Fig. 5a we present results for the integrated  $K^+d \rightarrow K^0pp$  cross section as a function of the kaon momentum in comparison to the available experimental in-

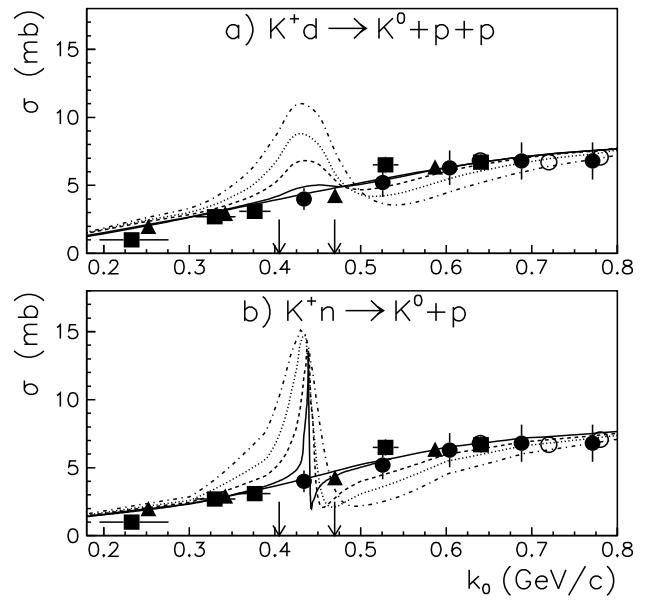


FIG. 5: Total  $K^+d \rightarrow K^0pp$  cross section as a function of the kaon momentum. The curves in a) show our full results for the original Jülich  $KN$  model without the  $\Theta^+$  resonance (solid line) and the variants with a  $\Theta^+$  and with different widths ( $\Gamma_\Theta = 1$  MeV - solid with bump; 5 MeV - dashed; 10 MeV - dotted; and 20 MeV - dash-dotted). The curves in b) correspond to a calculation for the reaction  $K^+n \rightarrow K^0p$  assuming that the neutron target is at rest. Data are from Refs. [4] (filled circles), Ref. [6] (squares), [8] (triangles) and [7, 9] (open circles). The vertical arrows indicate the range of kaon momenta corresponding to the smallest and the largest values found experimentally for the mass of the  $\Theta^+$  resonance.

formation. Again, it is evident that the data of Glasser et al. [8] and Damerell et al. [4], as mentioned above, provide the most restrictive constraints for the  $\Theta^+$  width. Thus, it is certainly fortunate that there are two independent measurements in the critical energy range. One can see from Fig. 5a that none of the model calculations with a  $\Theta^+$  width larger than 1 MeV is compatible with the data. Widths of 1 MeV or less can be certainly accommodated though we should say here that we did not explore the effect of such narrow widths in an actual model calculation.

If we disregard again the two data points from Refs. [8] and [4], respectively, there is a larger gap in the data base just at those energies where the  $\Theta^+$  is supposed to be located (the largest and smallest resonance masses reported so far are indicated by bars in Fig. 5) and that allows to fit in such a resonance with a width of  $\Gamma_\Theta \approx 5$  MeV without increasing the  $\chi^2/\text{dof}$  by more than 10%, cf. Table 1.

In order to illustrate the impact of the stationary neutron approximation we show here also calculations for the two-body reaction  $K^+n \rightarrow K^0p$ , cf. Fig. 5b. Comparing the two panels of the figure one can see to which extend the resonance is broadened by the Fermi motion of the

nucleons in the deuteron and by the integration over the three-body phase space.

In summary, we have investigated the impact of the  $\Theta^+(1540)$  resonance on the reaction  $K^+d \rightarrow K^0pp$  where experimental information is available at kaon momenta below 640 MeV/c. The calculation utilizes the Jülich  $KN$  model and extensions of it that include contributions from a  $\Theta^+(1540)$  state with different widths. The evaluation of the reaction  $K^+d \rightarrow K^0pp$  takes into account effects due to Fermi motion of the nucleons within the

deuteron and the final three-body kinematics. The comparison with existing data on differential and integrated cross sections suggests that there is no room for a  $\Theta^+$  resonance with a width of more than 1 MeV.

We appreciate discussions with W. Eyrich, C. Hanhart, A.W. Thomas and I. Zahed. This work was partially supported by the grant N. 447AUS113/14/0 by the Deutsche Forschungsgemeinschaft and the Australian Research Council.

---

- [1] For a recent review see e.g. E.Klempt, hep-ph/0404270.
- [2] D.K. Hong, Y.J. Sohn, I. Zahed, Phys. Lett. **B596** (2004) 191.
- [3] D. Melikhov, S. Simula, B. Stech, Phys. Lett. **B594** (2004) 265.
- [4] C.J.S. Damerell et al., Nucl. Phys. **B94** (1975) 374.
- [5] V.J. Stenger et al., Phys. Rev. **134** (1964) B1111.
- [6] W. Slater et al., Phys. Rev. Lett. **7** (1961) 378.
- [7] G. Giacomelli et al., Nucl. Phys. **B42** (1972) 437.
- [8] R.G. Glasser et al., Phys. Rev. **D15** (1977) 1200.
- [9] G. Giacomelli et al., Nucl. Phys. **B37** (1972) 577.
- [10] S. Nussinov, hep-ph/0307357.
- [11] R.N. Cahn and G.H. Trilling, Phys. Rev. **D69** (2004) 011501.
- [12] W. R. Gibbs, nucl-th/0405024.
- [13] R.A. Arndt, I.I. Strakovsky and R.L. Workman, Phys. Rev. **C68** (2003) 042201(R).
- [14] J. Haidenbauer and G. Krein, Phys. Rev. **C68** (2003) 052201(R).
- [15] A. Sibirtsev, J. Haidenbauer, S. Krewald, and U.-G. Meißner, hep-ph/0407011.
- [16] K. Hashimoto, Phys. Rev. **C29** (1984) 1377.
- [17] R. Büttgen et al., Nucl. Phys. **A506** (1990) 586.
- [18] M. Hoffmann et al., Nucl. Phys. **A593** (1995) 341.